Applied Mathematics and Statistics

Foundation Qualifying Examination Part B

in Computational Applied Mathematics

Summer 2017 (May)

(Closed Book Exam)

Please solve 3 out of 4 problems for full credit.

Indicate below which problems you have attempted by circling the appropriate numbers:

Part B: 1 2 3 4

NAME ____________________________________

Start each answer on its corresponding question page. Print your name, and the appropriate question number at the top of any extra pages used to answer any question. Hand in all answer pages.

Date of Exam: May 23, 2017

Time: 11:00 AM – 12:00 PM

B1.

a) Derive a Riccati equation that arises in the process of factoring a second order differential operator

$$\frac{d^2}{dx^2} + P_1(x)\frac{d}{dx} + P_0(x) = \left[\frac{d}{dx} + a(x)\right]\left[\frac{d}{dx} + b(x)\right].$$

b) Solve the corresponding Riccati equation to factor the following differential operator

$$\frac{d^2}{dx^2} + \frac{x^2+1}{x}\frac{d}{dx} + 2.$$

B2. Obtain a first order perturbative approximation $y(x) = y_0(x) + \varepsilon y_1(x)$ to the initial value problem with a small positive parameter ε

$$y'' + (1+\varepsilon)y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

Compare the behavior of the perturbative solution at large x with the exact solution. In which x-domain is this approximate solution valid?

B3. Consider the matrix $C = (A^T A)^{-1}$, where $A \in \mathbb{R}^{m\times n}$ with $\operatorname{rank}(A) = n$.

a) (5 points) Suppose the reduced QR factorization $A = QR$ is available. Show that $C = (R^T R)^{-1}$.

b) (5 points) What is the condition number of C in 2-norm in terms of the singular values of A?

B4. Consider $A \in \mathbb{C}^{m \times n}$ and $B \in \mathbb{C}^{n \times m}$.

a) (6 points) Show that $\begin{bmatrix} AB & 0 \\ B & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ B & BA \end{bmatrix}$ are similar to each other.

b) (4 points) Use the result in (a) to show that the nonzero eigenvalues of AB are the same as those of BA.